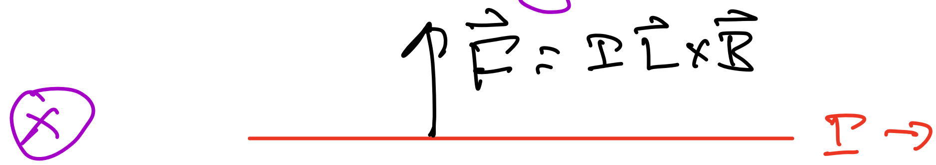
Force on current carrying wire in external $\vec{B}$



B into page everywhere

resulting force is up

we often see symmetry in physics

wire/current + $\vec{B}_{ext}$ ⇒ wire/motion

try:

wire/motion + $\vec{B}_{ext}$ ⇒ wire/current?

if you move a wire in an external B will you get a current induced in the wire?

Yes!

try: (move the magnetic field)

wire + $\vec{B}_{ext}$/motion ⇒ wire/current?

Yes again!

does $\vec{B}_{ext}$ have to be moving?

⇒ what if $\vec{B}_{ext}$ is increasing/decreasing?

does that induce a current in the wire?

Yes again!

so there's a relationship between $\frac{dB}{dt}$ and $I_{induced}$ in wire

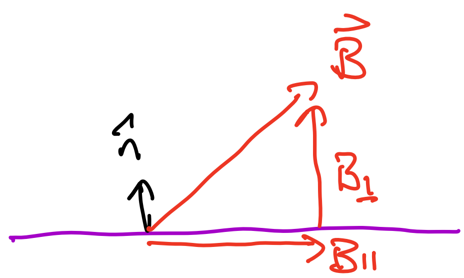
Further experimentation shows:

⇒ the change in magnetic flux is what causes currents to flow in wires

Remember flux is $\vec{B}$ integrated over surface

$\hat{n}$ is vector ⊥ ("normal") to surface

$\vec{B} \cdot \hat{n}$ is $B_\perp$



component of $\vec{B}$ parallel to surface does not go thru surface. Only $B_\perp$ does

flux $\Phi = \int_{surface} \vec{B}\cdot\hat{n}\,dA$ or write $d\vec{A} = \hat{n}\,dA$

$$= \int_{surface} \vec{B}\cdot\hat{n}\,dA = \int_{surface} B_\perp\,dA$$

Faraday: 1791-1867 English scientist

~1834 discovered law of induction:

$$\mathcal{E} = -\frac{d\Phi}{dt}$$ in a loop of wire

$\mathcal{E}$ = emf (like a voltage) that drives current around loop

Φ = magnetic flux thru loop surface

what does "–" sign mean?

Faraday's law says: a changing magnetic flux thru a loop will generate EMF to cause a current to flow

But which direction? current can go

clockwise or counterclockwise

$\Rightarrow \frac{d\Phi}{dt} \rightarrow \text{EMF} \rightarrow I_{\text{induced}}$

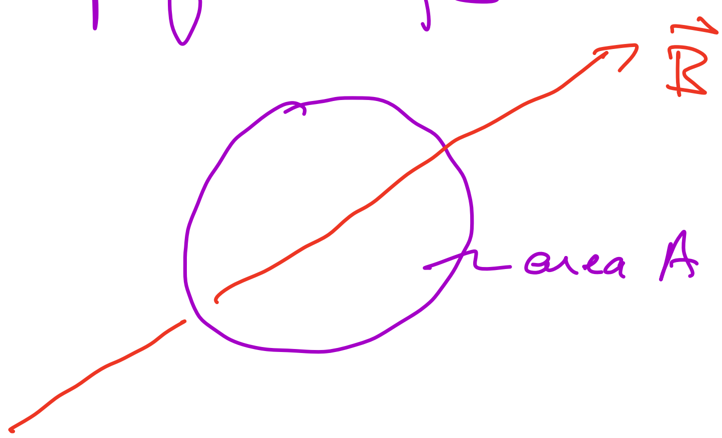
$I_{\text{induced}} \rightarrow B_{\text{induced}}$

(all currents generate mag fields)

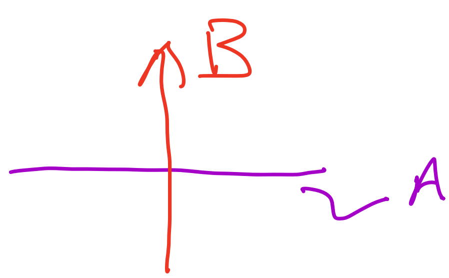
Faraday's law: induced B will oppose the <u>change</u> in Φ flux

$\Rightarrow$ nature does not want Φ to change!

ex: loop of wire, B thru loop!

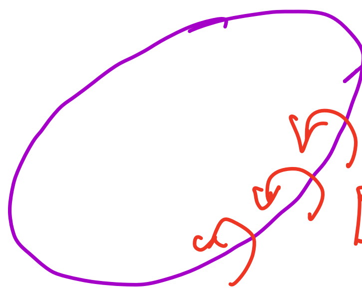


let B = constant and $\perp$ to A



$\Phi = B \cdot A$

- if B increases then $\frac{d\Phi}{dt} > 0$ increases
- this causes $I_{induced}$ which generates $B_{induced}$
- $B_{induced}$ goes thru loop as well



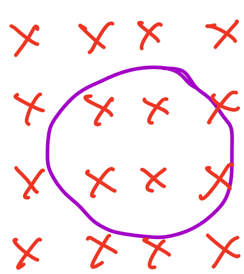
Faraday's law has the "$-$" sign:

$\Phi_{induced}$ by $B_{induced}$ thru loop will oppose change in total flux

$\Rightarrow$ if B is increasing & loop area is constant then Φ is increasing

$\Rightarrow$ $B_{induced}$ will be so that Φ doesn't increase (or increases slow as possible)

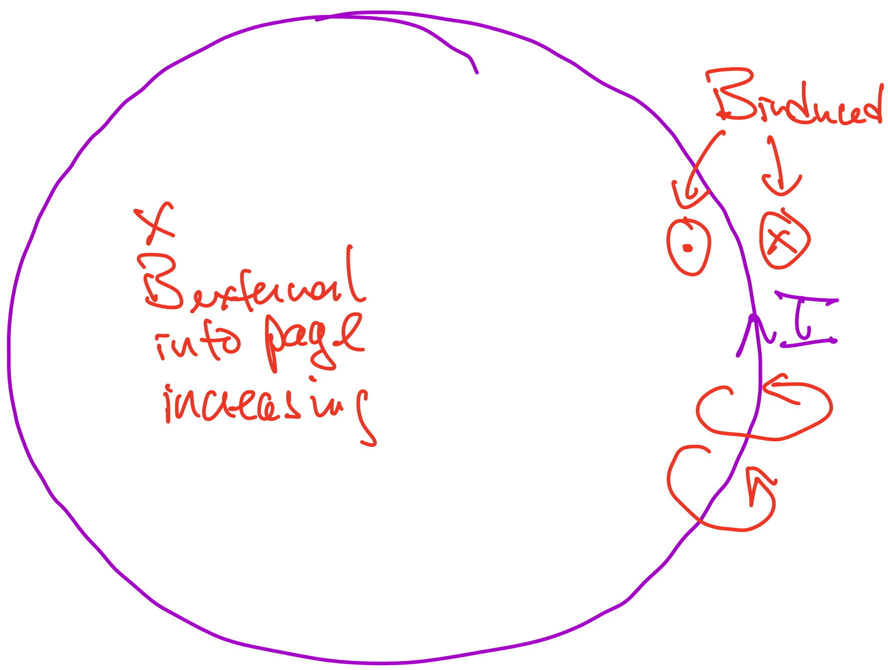
ex



B into page

if B ↗ into page then Φ ↗ downward

- want B induced to decrease change in Φ
- so B ind is out of page
- I induced counter clockwise by RHR produces B ind that is out of page inside the loop



If B is into page & decreasing then B ind will be out of page inside loop

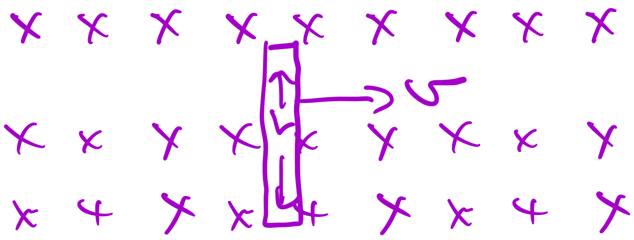
$\Rightarrow$ I ind will be clockwise

can use RHR:

- point thumb along direction of <u>change</u> in Φ
- fingers curl <u>opposite</u> to the EMF direction (& the current)

This opposition to the change in Φ is called Lenz's Law

Motional EMF



B into page, conducting bar, move to right w/vel v

charge in bar (conductor!) moves w/vel v so

Lorentz force $\vec{F} = q\vec{v} \times \vec{B}$ up

→ pos charge accumulates at top, neg at bot

causes E field & electric force is $q\vec{E}$

charges stop moving when $qE = qvB$

so $E = vB$

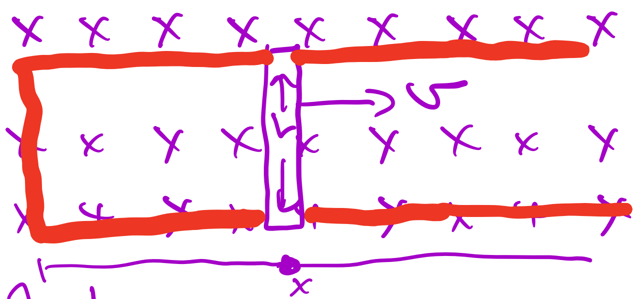
then there will be potential diff between top & bot: $V_{TOP} > V_{BOT}$ and for "capacitor"

$$V_{TB} = V_{TOP} - V_{BOT} \quad \text{and}$$

$$V_{TB} = EL$$

so $\frac{V_{TB}}{L} = vB$ or $V_{TB} = vBL$

now add conducting rails



area of loop increases:

$A = Lx$ but x increases

so dA change in A is $L\,dx$

and $v = dx/dt$ so $dx = v\,dt$

and $dA = L\,dx = vL\,dt$

constant over A

mag flux thru loop is $\phi = \int \vec{B}\cdot d\vec{A} = BA$

F's law:

$$\mathcal{E} = -\frac{d\phi}{dt} \rightarrow -\frac{d}{dt}BA = -B\frac{dA}{dt} = -B\frac{vL\cancel{dt}}{\cancel{dt}}$$

$$\mathcal{E} = vBL \quad (\text{ignore "}-\text{"!})$$

this causes current to flow, which generates $B_{induced}$

since ϕ is ↗, induced I will gen B_{ind} that keeps flux from changing, so B_{ind} opposes external B, so out of page

I_{ind} will be counterclockwise (up the bar) if the bar and rails have resistance R then magnitude of current is given by

$$I = \frac{\mathcal{E}}{R} = \frac{vBL}{R}$$

this also means energy will be dissipated in bar + rails:

$$P_{out} = I\mathcal{E} = \frac{vBL}{R} \cdot \mathcal{E} = \frac{(vBL)^2}{R}$$

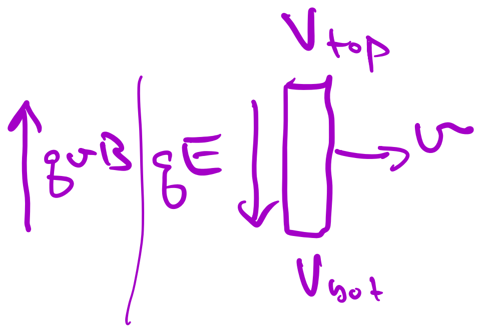
this power comes from the whoever is moving bar w/ velocity v

$$P_{in} = F_{in} v \quad \text{and} \quad F_{in} = F \text{ current in } B \text{ field}$$
$$= ILB$$

so $P_{in} = ILB \cdot v = I \cdot vBL = I\mathcal{E}$ ✓

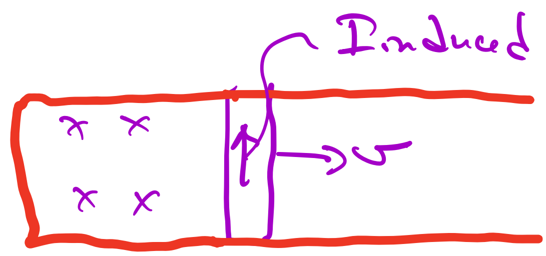
note: these give "same" answer for voltage (no rail) & $\mathcal{E}$ (rail)

no rail



$V_{TB} = vBL$

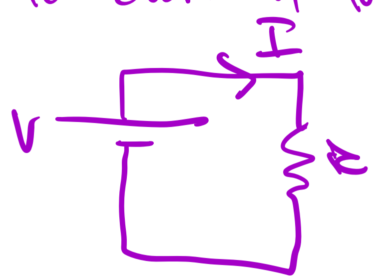
rail



$\mathcal{E} = vBL$

this does <u>not</u> mean there's a potential diff for rail, top & bot of bar!

note: current for battery loops flow higher to lower V



so for rail, current is up but that does <u>not</u> mean $V_{Bot} > V_{Top}$

Battery: V generates E field inside wire that pushes charge along

Rail: what's pushing is Lorentz force qvB so no internal $\vec{E}$ so no ΔV

Rail guns

"muzzle velocity" $v \sim 1200\,m/s$ modern high-velocity cartridges

$$1200\,\frac{m}{s} * \underbrace{\frac{3.28\,ft}{m} * \frac{1\,mi}{5280\,ft} * \frac{3600\,s}{hr}}_{2.24\,\frac{mph}{m/s}} = 2680\text{ mph}$$

Mach 1 speed of sound $\sim 343\,m/s * 2.24\,\frac{mph}{m/s} = 770\,mph$

So gun velocity $\sim$ mach 2

Armor penetrating velocity for tank gun $\sim 1700\,m/s$

$1700\,m/s * 2.24\,\frac{mph}{m/s} = 3800\,mph \sim$ Mach 5

can't go much faster w/ chemical propellants

Rail gun: use electromagnetics

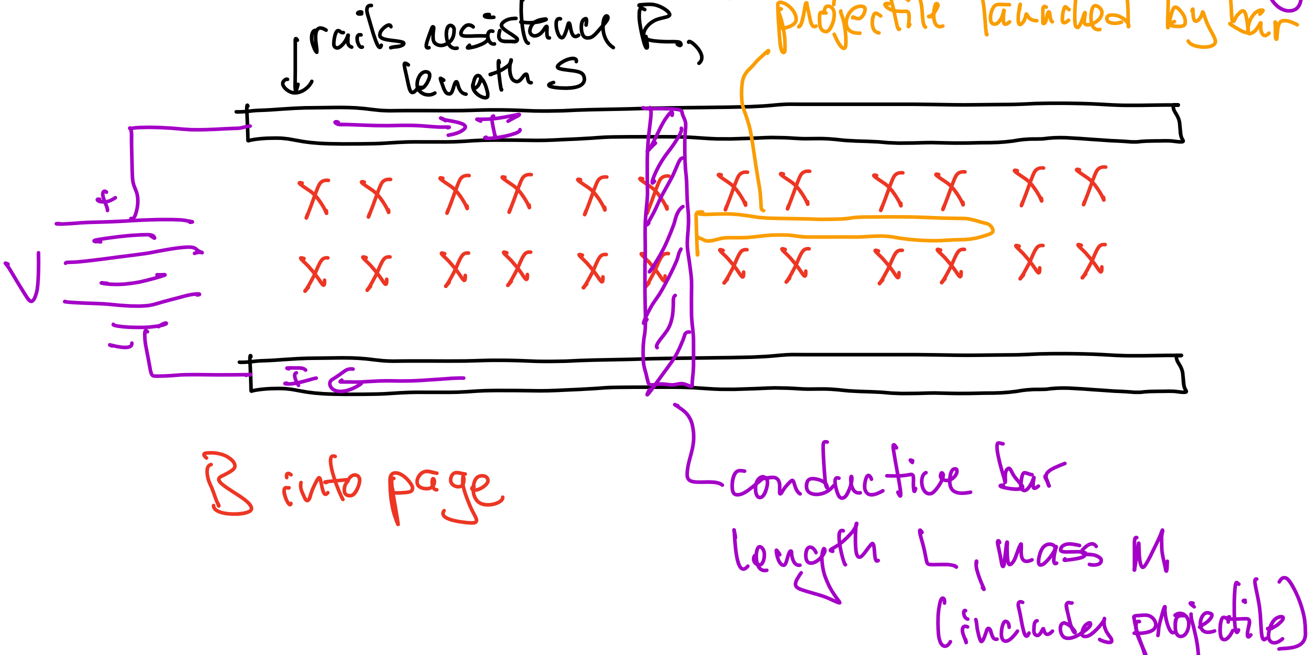
Motional EMF: velocity generates EMF & current

so expect current will generate velocity

Rail gun

motional EMF: you supply velocity to bar, induces a current

rail gun: you supply current, induces velocity!



- battery voltage pushes current clockwise around the loop: $I = V/R$
- current goes <u>DOWN</u> the bar
- Force on bar w/current: $F = ILB$ to the right
- force $F = ILB = ma$ acceleration
- (from kinematics: $2as = v^2 - v_i^2$
  s = total distance accelerated
  $v_i = 0$ starts from rest

$\therefore \quad a = \frac{v^2}{2s} \Rightarrow ma = \frac{mv^2}{2s} = ILB$

$$\boxed{I = \frac{mv^2}{2sLB}}$$

if we want $v = \text{mach } 7 = 340\frac{m}{s} * 7 \cong 2400\, m/s$

let $m = 5\text{ kg}$ (~10 lbs)

$s = 5\text{ m}$ long rail gun

$B = 1\text{T}$ (very large!)

$L = \frac{1}{2}\text{m} \sim 18\text{ in}$ long

then to get to Mach 7:

$$I = \frac{5\text{ kg} * (2400\, m/s)^2}{2 \cdot 5\text{m} \cdot \frac{1}{2}\text{m} \cdot 1\text{T}} = 5.8\times10^6\text{A} = 5.8\text{MA}$$

$V = IR \sim 10$ MVolts!

This is a very large ~~ba~~ battery

with 5.8×10^6 A, acceleration is

$$a = \frac{ILB}{m} = \frac{5.8\times10^6 * \frac{1}{2}\text{m} * 1\text{T}}{5\text{kg}} \cong 0.6\times10^6 \frac{m}{s^2}$$

time to get to $v = 2400\, m/s$ is given by $v = at$

so $t = \frac{v}{a} = \frac{2400}{0.6 \times 10^6} = 4.2 \times 10^{-3}\,s = 4.2\,ms$!

very fast !!

How does Faraday's law fit in?

The change in Φ is the same as before:

$$\frac{d\Phi}{dt} = BLv$$

of course v is increasing (its accelerating!)

lets use the average velocity $\bar{v} = \frac{1}{2}(v_f - v_i)$

$= \frac{1}{2} v$

$= 1200\,m/s$

then the induced EMF is

$$\mathcal{E}_{induced} = BLv$$

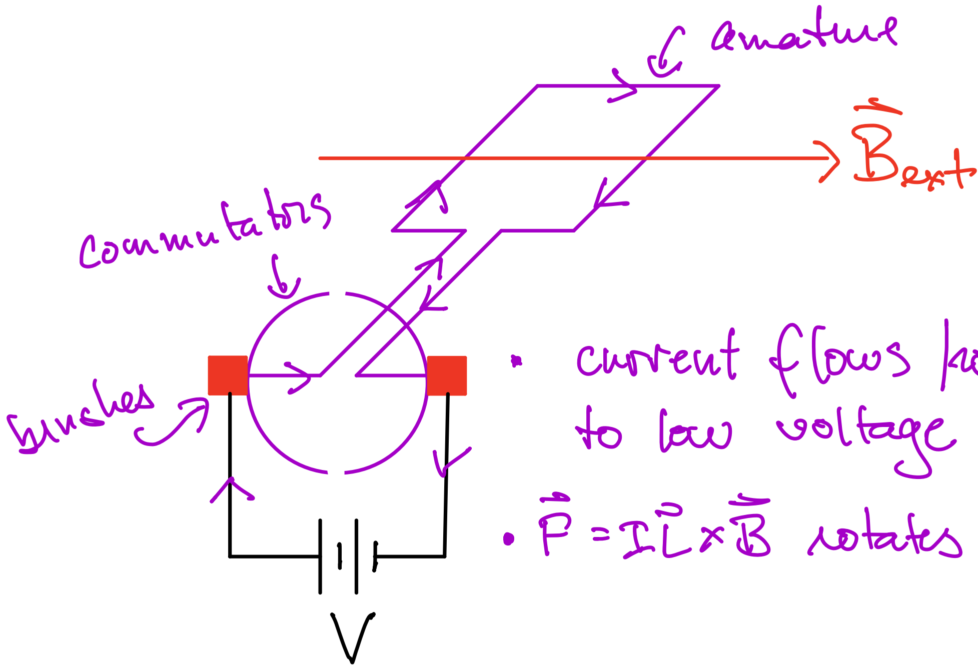
$$= 1T \times \frac{1}{2}m \times 1200\frac{m}{s}$$

$$= 600\,V$$

this will be small compared to the huge voltage needed to power the rail gun!

Alternator

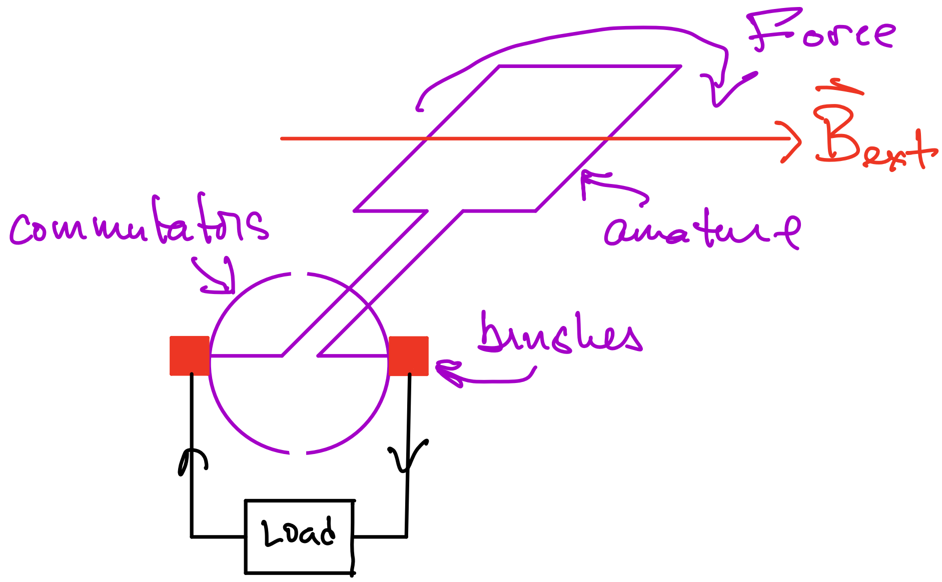
Chapter 27 : DC motor



- current flows from high to low voltage
- $\vec{F} = I\vec{L} \times \vec{B}$ rotates loop

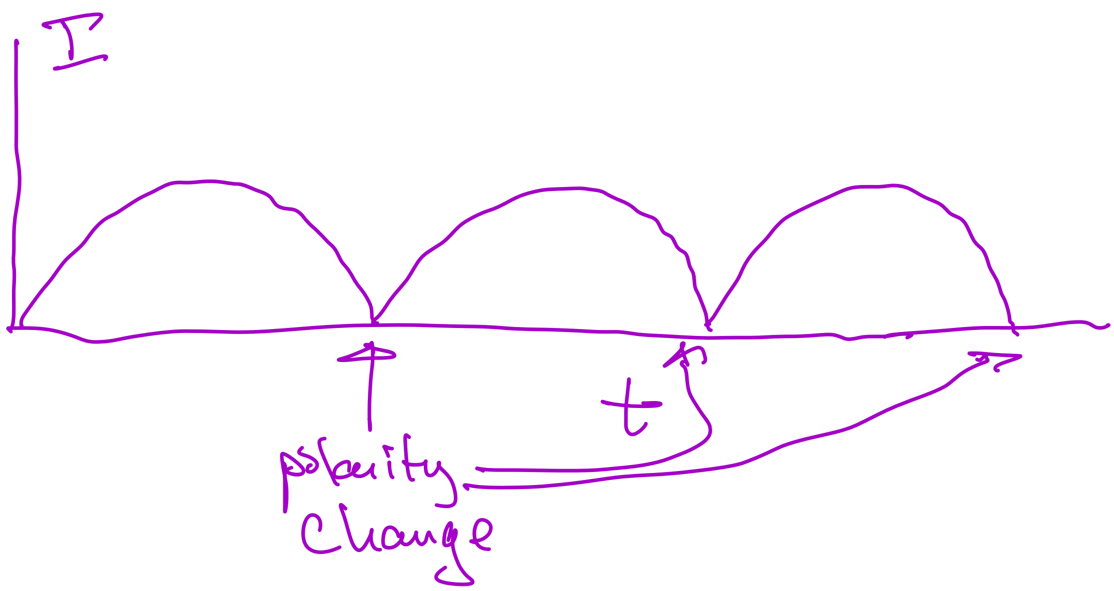
- when loop is rotated all the way, brushes & commutator cause current to change direction causing another force, more rotation, etc.

$\Rightarrow$ instead of using current to drive motion, input motion to generate a current

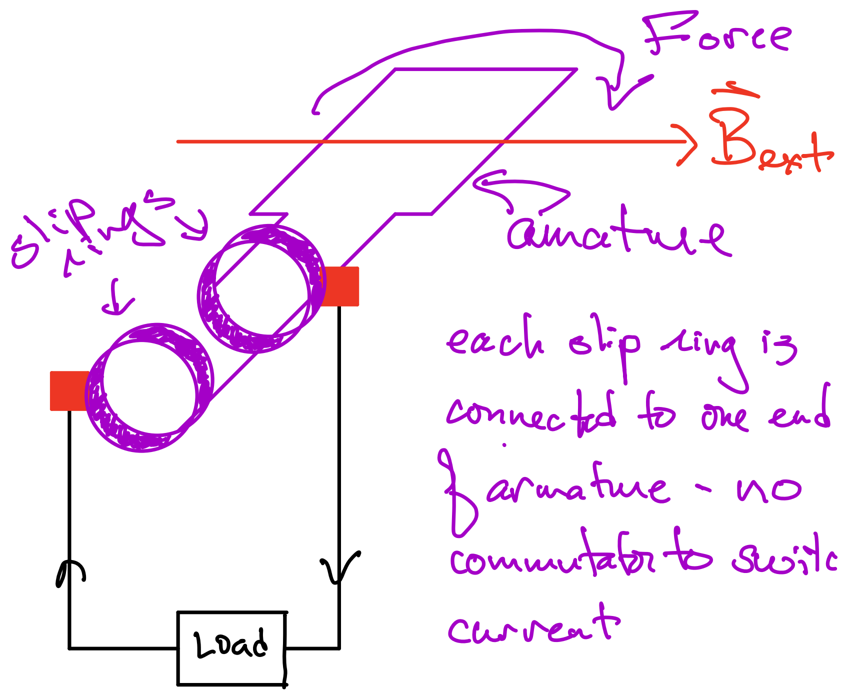


currents will flow thru the load, there & will be a voltage difference across it

⇒ note that as armature rotates, current will always flow in same direction thru load just like in DC motor the voltage source always pushes current in same direction

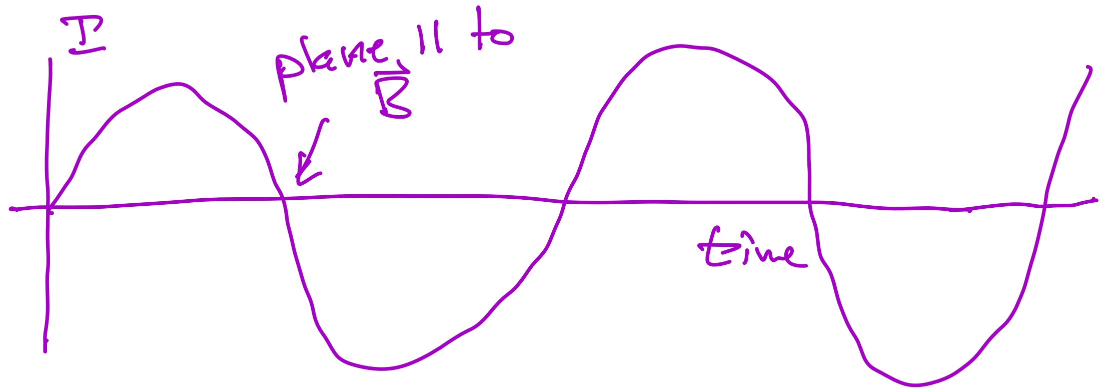


Alternator - clever variation on generator

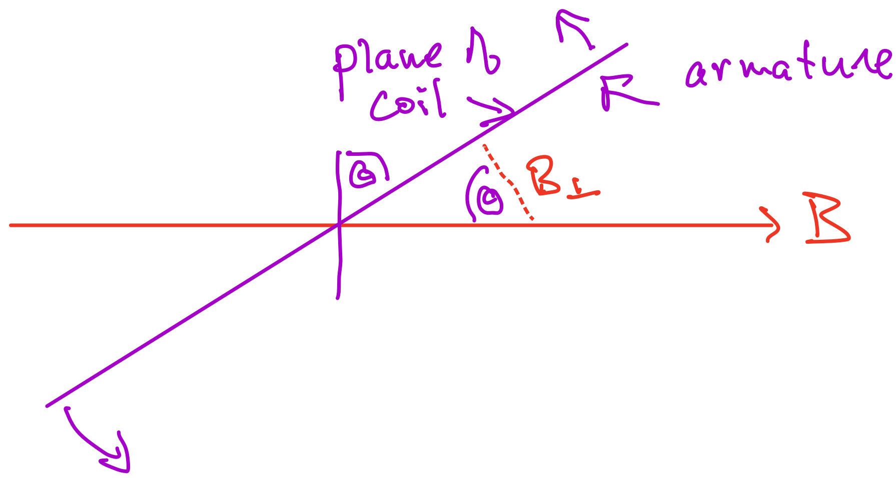


each slip ring is connected to one end of armature - no commutator to switch current

current will switch direction as armature rotates



as plane of armature rotates, Φ will change:



flux will be $B_\perp \cdot A = BA\cos\theta$

by Faraday's law: $\mathcal{E} = -\frac{d\Phi}{dt} = -BA\frac{d}{dt}\cos\theta$

$\frac{d\cos\theta}{dt} = -\sin\theta \cdot \frac{d\theta}{dt}$

$\frac{d\theta}{dt}$ is the "angular frequency"

in one period T, θ goes thru 2π radians

so $\frac{d\theta}{dt} = \omega = \frac{2\pi}{T}$

if $\omega =$ constant then $\theta = \omega t$

so EMF $\mathcal{E} = BA\omega\sin\omega t$

this allows you to generate any voltage by varying how fast you spin it (ω)

Generator vs alternator?

commutators, amature, brushes!

Armature: housing for wiring that carries current
Commutator: reverses current on DC generator/motor
Brushes: connects spinning parts to stationary
Stator: stationary part

- Generators produce DC which is what cars need
  DC comes from using split commutator

- Generators use split rings & brushes so wear out quickly
- Alt's have armature at rest in stator & field windings move.
  Can use electromagnets w/ iron core to amplify B, less current flows thru brushes.

  Gen's have field windings at rest so motor can turn heavy magnets but still need commutator to change polarity of field problem:

car generator turns at ~800 RPM when idling
Alternator has coil of 300 turns
loop is ~ 5cm x 8cm
Want $\mathcal{E}$ to vary w/ amplitude ~30V
find B

$\mathcal{E} = NBA\omega \sin\omega t$ so amplitude is
$NBA\omega$

$30 = 300 \cdot B \cdot 40\text{cm}^2 \cdot \omega$

$f = 800 \frac{\text{cycles}}{\text{min}} * \frac{1\text{min}}{60\text{sec}} = 13.3/\text{sec}$

$\omega = 2\pi f = 83.8 \text{ rad/sec}$

so $30 = 300 \cdot B \cdot 40\text{cm}^2 * \frac{1\text{m}^2}{(100\text{cm})^2} * 83.8$

$= 100.5 B$

$B = 30/100.5 = 0.3T$ pretty big field

note they can change $\mathcal{E}$ amplitude by varying current in windings that produce external field

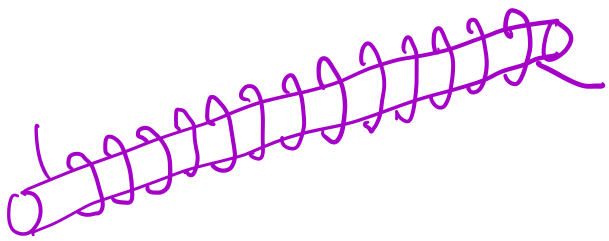
Induced E fields

we get EMF when flux changes: $\mathcal{E} = -\frac{d\Phi_m}{dt}$

and $\Phi = $ "B" $\cdot$ "A" $\int \vec{B}\cdot d\vec{A}$

and we've let area change

now let B change



solenoid, length L, N loops, $n = N/L$ turns/length

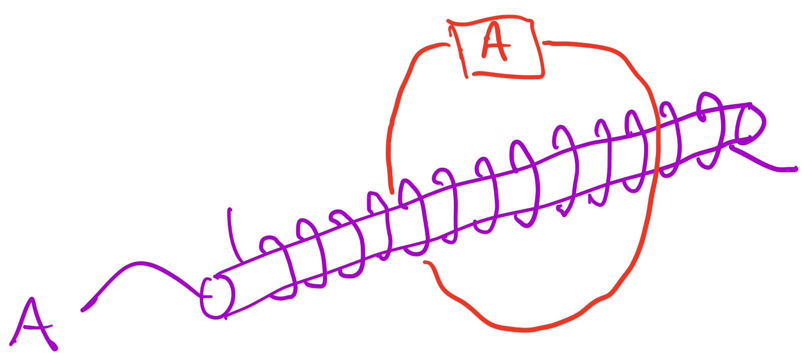
connect to power source, current I_{sol} flows

$B = \mu_0 n I_{sol}$ inside solenoid

now let power source vary so that $\frac{dI}{dt} \neq 0$

then $\frac{dB}{dt} = \mu_0 n \frac{dI_{sol}}{dt}$ (B varies)

add 2^{nd} loop around (or 2^{nd} coil)



flux thru outer coil $\int \vec{B} \cdot d\vec{A}$

assume $B \neq 0$ inside solenoid, $B = 0$ outside

$$\therefore \int_{\text{outer coil}} \vec{B} \cdot d\vec{A} = B \cdot A_{\text{solenoid}} \leftarrow \text{not } A_{\text{coil}}$$

so $\mathcal{E}$ around outer coil

$$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{d}{dt}(B \cdot A) = -A_{\text{solenoid}} \cdot \frac{dB_{\text{solenoid}}}{dt}$$

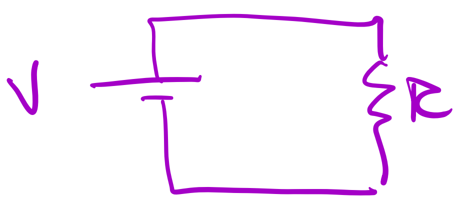
$$= -\mu_0 n A \frac{dI_{sol}}{dt}$$

outer coil has resistance R_{coil}

→ will have current measured by galvanometer:

$$I_{coil} = \frac{\mathcal{E}}{R_{coil}} = \frac{\mu_0 n A_{sol}}{R_{coil}} \frac{dI_{sol}}{dt}$$

EMF is <u>not</u> like a voltage!



current around circuit gains no net energy (conservative)

EMF must come from an E field that generates force $F = qE_{ind}$ that pushes charge around

this E_{ind} comes from <u>changing</u> B-field

and <u>not</u> from external electric charges!
so is <u>not</u> conservative

remember $\Delta U = \int \vec{F} \cdot d\vec{l}$ work = change in energy

and $\Delta V \equiv \frac{\Delta U}{q}$ change in voltage

so $\Delta V = \int \underbrace{\frac{\vec{F}}{q}}_{\vec{E}} \cdot d\vec{l} = \int \vec{E} \cdot d\vec{l}$

battery

around a loop, E (static from charges) is conservative

so $\Delta V = \oint \vec{E}_s \cdot d\vec{l} = 0$

but for induction $\Delta V \equiv \mathcal{E} = \oint \vec{E}_i \cdot d\vec{l}$

so by Faraday:

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_m}{dt} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$$

this relates E to <u>changing</u> B & is one of

maxwell's laws

now apply cons of energy:

charge Δq gains energy $\Delta U = \Delta q \cdot \mathcal{E}$
going around loop

this induces current I_{ind} in larger loop

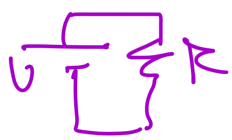
$$I_{ind} = \frac{\mathcal{E}}{R_{coil}} = \frac{\mu_0 n A}{R_{coil}} \frac{dI_{sol}}{dt}$$

and since outer coil is conductor w/ resistance, will dissipate energy $I_{ind}\,\mathcal{E}$

this energy reduces energy of Δq around loop

if you added a load instead of galvanometer, you need more $\mathcal{E}$ to drive it.

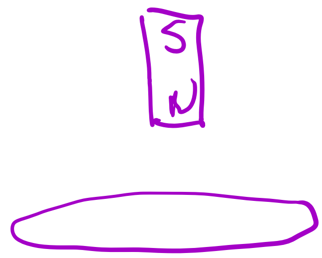
so is $\mathcal{E}_{ind}$ really a voltage? <u>no</u>!

for battery 

current flows because the battery generates E-field inside the wire

for Faraday's law, $\mathcal{E}_{ind}$ causes I_{ind} really due to the Lorentz force $\vec{F} = q\vec{v} \times \vec{B} \Rightarrow \vec{B}$ is moving the charges!

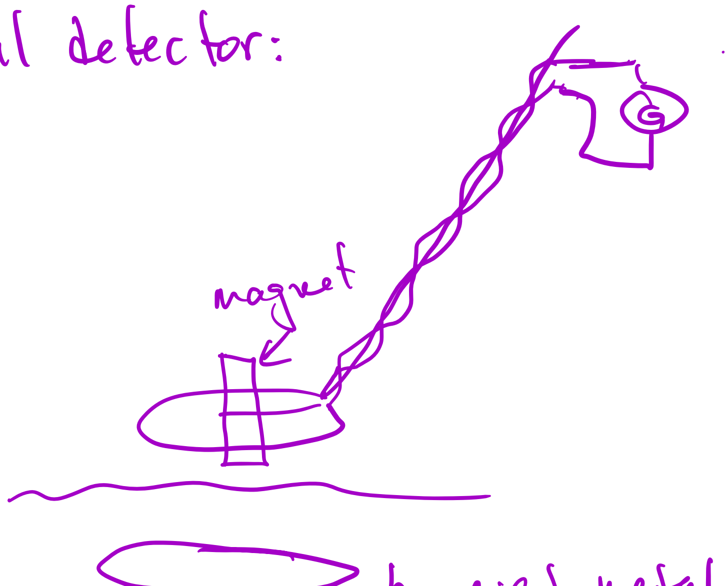
Eddy currents



flat conducting disk.

- as B changes thru face, currents flow
- can flow anywhere but always in loop
- these currents cause B_{ind} that can be detected

Metal detector:



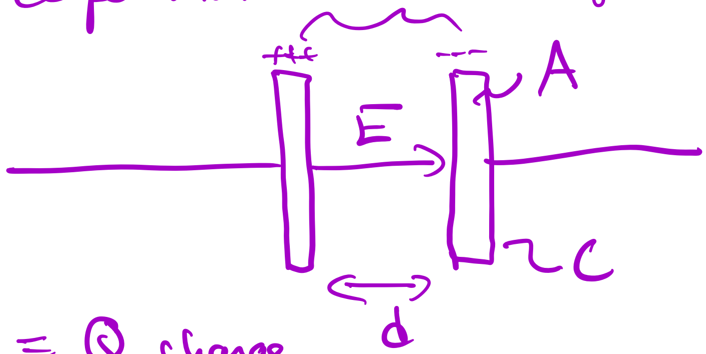
buried metal conductor

magnet moves over metal, causes Eddy currents that cause induced magnetic field B_{ind}

B_{ind} changes thru loop connected to galvanometer when over metal

Displacement current:

Charge capacitor. V voltage across plates



$C = \frac{\varepsilon_0 A}{d} = \frac{Q}{V}$ $\frac{\text{charge}}{\text{voltage}}$

while charging, charge accumulates on plates
equal Q on + and $-Q$ on − plate

generates E field across: $E = V/d$

so $V\frac{\varepsilon_0 A}{d} = Q$ and $\frac{V}{d} = E$

so $Q = \varepsilon_0 \underbrace{AE}_{\text{electric flux } \Phi_E}$

so $Q = \varepsilon_0 \Phi_E$

$\underbrace{\frac{dQ}{dt}} = \varepsilon_0 \frac{d\Phi_E}{dt}$ as current changes

looks like a current that "flows" as long as $\frac{dQ}{dt} \neq 0$

call this "displacement current" I_D

so $$I_D = \varepsilon_0 \frac{d\Phi_E}{dt}$$

I_D is the same as $I = \frac{dQ}{dt}$ flowing thru the circuit! $\Rightarrow$ its just the equivalent to what's happening inside the capacitor

by Amp's law, $\vec{B}$ would loop around wire & current thru

so $$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{in}$$

and $I_{in} = I$ in wire & I_D in capacitor

so $$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I + \varepsilon_0 \frac{d\Phi_E}{dt} \right)$$

Maxwell's equations:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

closed surface

$$\oint \vec{B} \cdot d\vec{A} = 0$$ no magnetic charges

Ampere's law, displacement current

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{in} = \mu_0 I + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt}$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$ Faraday's Law

if no sources: $Q=0, I=0$

gives

$$\oint \vec{E} \cdot d\vec{A} = 0$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{B} \cdot d\vec{l} = \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt}$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$